Exercise 10

For the following exercises, points $P(1.5, 0)$ and $Q(\phi, y)$ are on the graph of the function $f(\phi) = \cos(\pi\phi)$.

Complete the following table with the appropriate values: y-coordinate of Q, the point $Q(\varphi, y)$, and the slope of the secant line passing through points P and Q. Round your answer to eight significant digits.

ϕ	y	$Q(\phi, y)$	m_{sec}
1.4	a.	e.	i.
1.49	b.	f.	j.
1.499	c.	g.	k.
1.4999	d.	h.	l.

[TYPO: Replace φ with ϕ.]

Solution

If $\phi = 1.4$, then $y = \cos(\pi \cdot 1.4) \approx -0.30901699$, which means $Q(1.4, -0.30901699)$ and

$$m_{\text{sec}} \approx \frac{-0.30901699 - 0}{1.4 - 1.5} \approx 3.0901699.$$

If $\phi = 1.49$, then $y = \cos(\pi \cdot 1.49) \approx -0.031410759$, which means $Q(1.49, -0.031410759)$ and

$$m_{\text{sec}} \approx \frac{-0.031410759 - 0}{1.49 - 1.5} \approx 3.1410759.$$

If $\phi = 1.499$, then $y = \cos(\pi \cdot 1.499) \approx -0.0031415875$, which means $Q(1.499, -0.0031415875)$ and

$$m_{\text{sec}} \approx \frac{-0.0031415875 - 0}{1.499 - 1.5} \approx 3.1415875.$$

If $\phi = 1.4999$, then $y = \cos(\pi \cdot 1.4999) \approx -0.00031415926$, which means $Q(1.4999, -0.00031415926)$ and

$$m_{\text{sec}} \approx \frac{-0.00031415926 - 0}{1.4999 - 1.5} \approx 3.1415926.$$

For $f(\phi) = \cos(\pi\phi)$, the slope of the secant line passing through P and Q gets closer and closer to π as ϕ gets closer and closer to 1.5.